Audio Engineering Society

Convention Paper

Presented at the 141st Convention
2016 September 29–October 2 Los Angeles, USA

Design of Free-Standing Constant Beamwidth Transducer (CBT) Loudspeaker Line Arrays for Sound Reinforcement

D. B. (Don) Keele, Jr.[1]

[1] *DBK Associates and Labs, Bloomington, Indiana 47408, USA*
DKeeleJr@Comcast.net

ABSTRACT

This paper presents design guidelines for choosing the parameters of a free-standing CBT line array including its physical height, circular arc angle, location, and downward pitch angle to appropriately cover a single 2D straight-line audience sound-reinforcement listening region with direct sound. These parameters and conditions include: 1) array circular-arc angle and its associated beamwidth, 2) array height and low-frequency beamwidth control limit, 3) array mounting location which includes its height and setback from the front of the seating plane, and 4) the array's on-axis aiming location and associated downward pitch angle. These parameters are particularly easy to determine in advance for a CBT line array because of the extreme uniformity of its sound field with both frequency and distance, and its inherent constant-directivity characteristics.

This paper describes a design scenario that allows the designer to easily choose these system parameters to optimize the direct-field coverage in the prescribed straight-line seating region while minimizing the use of sound-system design and prediction software. The design technique forces the SPL at the front and rear of the listening region to be equal by aiming the array at the rear of the listening region and then choosing its beamwidth (and its associated off-axis rolloff) to provide this front-rear SPL equality. The SPL and frequency response at intermediate points of the covered region are then set by the inherent well-behaved off-axis rolloff of the CBT array.

1. INTRODUCTION

1.1. General Comments

CBT or constant beamwidth transducer theory is based on un-classified military under-water transducer research done in the late 1970s and early 80s was applied to loudspeaker arrays by Keele in a series of nine AES technical papers between 2000 and 2015 [7-15]. CBT arrays provide wide-band extremely constant beamwidth and directivity behavior with virtually no side lobes.

This paper describes a simplified CBT array design procedure and aiming scenario that essentially optimizes the array's direct-field coverage of an arbitrary straight-line 2D listening region. The method depends on aiming the array at the farthest listening point and then choosing the array's beamwidth to force the SPL at the nearest point to equal the SPL at the farthest point. The SPL and frequency response at intermediate points in the coverage region are then very well behaved due to the extreme uniformity of the CBT array's off-axis behavior.

In effect, the strongest output of the array is aimed at the farthest listening point while the nearest point is covered by the array's reduced off-axis output, thus equalizing near-far levels. The array's height, arc angle, and beamwidth are chosen appropriately to make this scheme possible. This method is similar to previous techniques used to aim constant directivity horns but is much superior because of the extremely well-behaved and predictable broad-band off-axis behavior of the CBT array and the design flexible afforded by the capability to tailor the beamwidth of the CBT array by changing the array's height and arc angle.

The method is equally suitable for both outside venues where only direct-field coverage is important, but also for indoor situations where room acoustics comes into play because the CBT array is one of the few acoustic radiators that inherently exhibit broadband constant directivity.

1.2. Traditional Design Techniques

Traditional sound reinforcement design techniques depend heavily on aiming and acoustic modeling software such as EASE and EASE Focus [1], Odeon [2], Modeler [3], and JBL's Line Array Calculator [4]. Often these sophisticated modeling programs are required so that designs get around the inherent limitations of the loudspeaker array itself that is being modeled.

Most array configurations have a difficult time providing both good direct field coverage of the audience region while simultaneously providing well-behaved energy radiated in other directions not directed at the audience. CBT arrays are different. They are inherently constant-directivity/constant-coverage and can simultaneously provide flat direct-field coverage of an audience area while at the same time provide extremely well-behaved out of beam coverage with extremely low lobes.

To attain even coverage of the listening region with traditional arrays, it typically takes much experimentation and trial and error with the modeling program to determine the configuration of the array and its aiming to suitably cover the audience area.

The array configuration and setup is often very complicated and includes many variables such as array shape, splay angles, number of boxes, the vertical coverage of each box, gains, delays, and local/global equalizations that have to be juggled to attain the desired coverage. Often optimization techniques are employed to determine the best values for all these many variables [5 – 6].

One unfortunate result of this conventional design process is that in the process of optimizing the direct field coverage of the audience area, the coverage in other areas is frequently neglected. A specific array may provide excellent direct-field coverage of the audience region but the total system may exhibit frequency dependent directivity and erratic out-of-beam coverage with frequency that contributes to a poor power response and lower system gain before feedback.

1.3. Comprehensive Analysis of SPL Error and Other Parameters

Before presenting the design techniques, this paper accomplishes a comprehensive analysis to determine all the acceptable source locations in a large representative rectangular region. Parameters analyzed include: SPL error, near-far distance ratio and required source beamwidth,

1.4. CBT Arrays Simplify Design Process

CBT arrays considerably simplify this design process because of their extremely well-behaved polar and directivity behavior with both frequency and distance [7 – 15]. The constant directivity nature of the CBT array makes it extremely easy to provide coverage in arbitrary audience regions in both interior and outdoor spaces because CBT arrays can simultaneously provide flat direct-field coverage of the audience region and extremely well behaved out-of-beam coverage with extremely low side lobes.

1.5. Symmetrical vs. Asymmetrical Coverage Patterns

Note that the design process described in this paper differs significantly from the design process described in [13]. The CBT arrays used here are the arrays described in the original series of CBT papers [7-12] that provide a symmetrical coverage pattern. Reference [13] describes an alternate coverage design method that modifies the standard CBT symmetrical coverage pattern into an asymmetrical pattern that more closely matches the audience region.

2. CBT LINE ARRAY VERTICAL POLAR RESPONSE

This section briefly describes CBT line array theory and the vertical polar response provided by a CBT line array.

2.1. Brief Overview of CBT Arrays

As described in [7], a general CBT line array is formed by a continuous circular-arc source of arbitrary angle and size with special frequency-independent shading provided by a Legendre function. The shading controls the amplitude of the source as a function of its angle with the level maximum in the center and tapering off on either end of the arced source. The theory can be extended to discrete arrays of point sources like loudspeakers. Figure 1 illustrates a CBT circular-arc line array with frequency-independent Legendre shading.

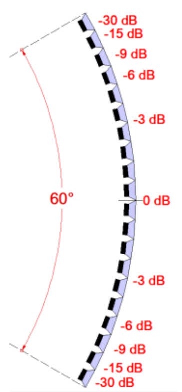

Fig. 1. Illustration of a free-standing loudspeaker CBT 60° circular-arc line array with Legendre shading. The frequency-independent shading levels are maximum in the center of the array and tapers off towards the outside edges of the array.

Thus formed, the line array provides a broadband symmetrical directional coverage whose beam pattern and directivity is essentially independent of frequency above a certain beamwidth control cutoff frequency, and also changes very little with distance from the source.

As quoted in [16]: “The surface distribution as well as the pressure distribution at all distances out to the far field is approximately equal to the surface distribution. Thus in a sense, there is no nearfield.”

2.2. Vertical Polar Response

The vertical polar response of a free-standing CBT line array at all distances closely follows the Legendre shading function that is used to shade the amplitude of the source [13]. Figure 2 shows the simplified power series approximation of the Legendre shading developed in [7, Eq. 3]. Because the far field polar pattern essentially follows the shading function, Figure 2 indicates that the resultant far field beamwidth (-6 dB) is approximately equal to 64% of the arc angle at the points where the shade level drops to 0.5 (6 dB down).

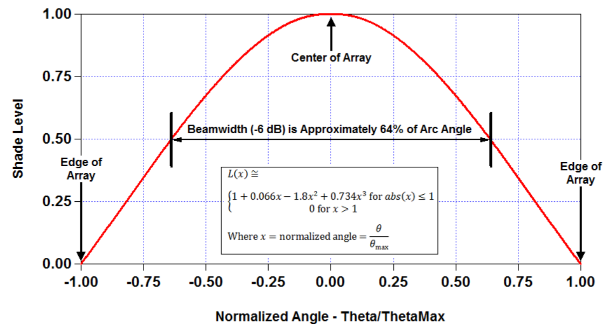

Fig. 2. Power series approximation of Legendre shading. The shade level is unity in the center of the array and smoothly falls to zero at the outside edges. Note the linear (non dB) vertical scale.

Reference [10] presents a more practical alternate shading function that is truncated at the -12 dB down point (0.25) where the 6 dB down beamwidth is then approximately 75% of the arc angle. Effectively, this alternate shading function eliminates those sources that are severely attenuated near the outside of the array without significantly compromising the arrays polar response. All the designs described in this paper all conform to this alternate truncated shading function.

3. SIMULATOR DESCRIPTION

This section briefly describes this paper’s simulator that predicts the 2D vertical plane coverage of a CBT array. The simulator was written using the program Igor Pro 6 from WaveMetrics (http://www.wavemetrics.com/) using point-source “Huygens” computer modeling techniques [17]. The simulator draws heavily from simulation code used in my previous CBT papers [7 – 15].

A custom simulator was required to calculate the graphical data displayed in this paper. Note that this simulator is not required to design the array and the coverage predictions described in this paper.

3.1. CBT Array Setup Parameters

In a typical CBT array sound reinforcement 2D coverage design, the following parameters are specified to cover a specific audience region:

1. Array circular-arc angle and its associated beamwidth angle,
2. Array height and its associated beamwidth low-frequency control limit (array LF cutoff frequency),
3. Array mounting location which includes its height and setback from the seating plane,
4. The array's on-axis aiming location and associated downward pitch angle.

3.2. Simulator Graphical Input and Output

Fig. 3 shows the simulator user interface with the entered and calculated data. The simulator's three data output graphs are shown in Figs. 4, 5, and 6.

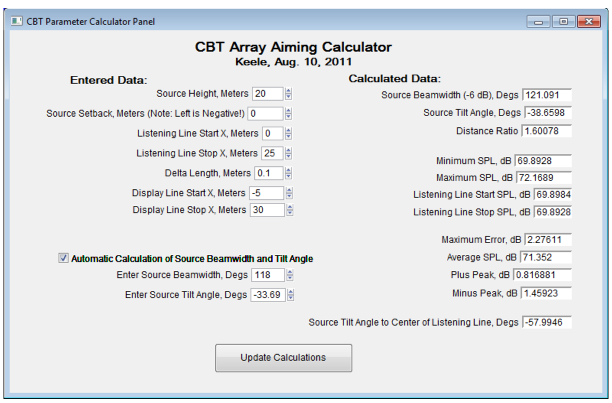

Fig. 3. User interface of CBT array aiming simulator with entered and calculated data.

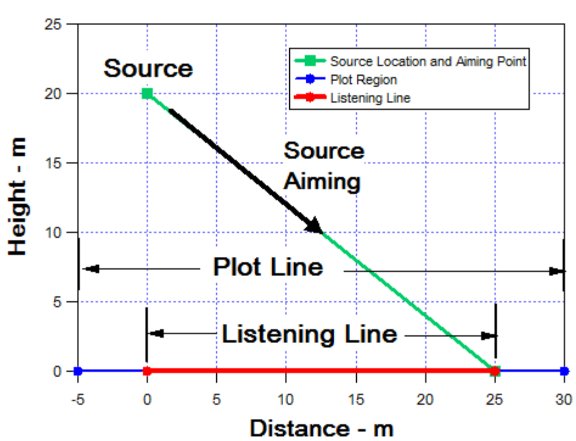

Fig. 4. Simulator output showing simulation region with source location and aiming. The listening line is the straight-line region where the SPL is calculated. For this plot the source is 20 m high and located over the start of the 25 m listening line.

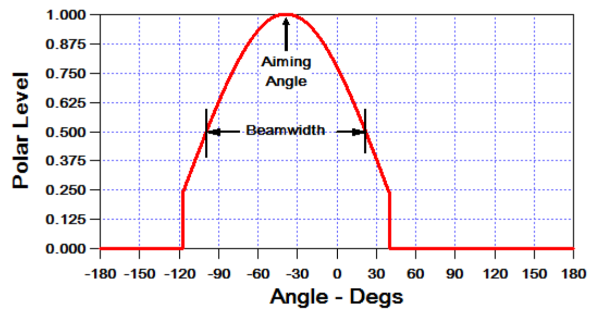

Fig. 5. Simulator output showing plot of the CBT directional coverage beam on rectangular graph. Note linear (non dB) scale on left. A 0.25 (-12 dB) truncated shading is assumed. This plot shows the coverage beam for the array mentioned in Fig. 4. The beamwidth is about 120 degs and is aimed down at an angle of about 40 degs.

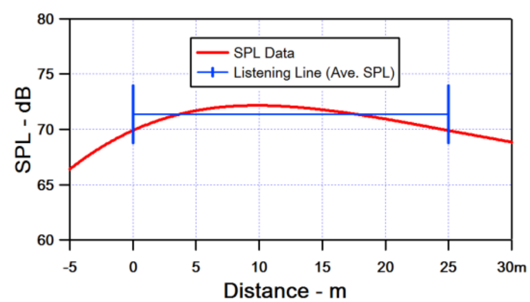

Fig. 6. Simulator output showing plot of SPL on listening or coverage line (red) with average SPL line plotted (blue horizontal line). Bold vertical lines denote the start and stop of the coverage line. A maximum error of about 2.3 dB is illustrated for the array data of Fig. 4, i.e. roughly ±1.25 dB around the average SPL line.

4. ANALYSIS OF SPL ERROR, NEAR-FAR DISTANCE RATIO, AND REQUIRED BEAMWIDTH VS. SOURCE LOCATION

This section presents the results of a comprehensive set of analyses that determines acceptable locations for CBT array sources to cover a specific straight listening line using the guidelines described in this paper. Criteria evaluated include maximum SPL error, near-far distance ratios, and required array beamwidth.

The assumption for all these simulations is that the CBT array is aimed at the farthest point of the listening line and that the beamwidth of the source is chosen to force the SPL's at the nearest and farthest points of the listening region to be equal.

To illustrate the many possible source locations for a possible coverage situation, a very large 200 m by 200 m rectangular region was analyzed. Figure 7 shows the 2D region analyzed for possible source locations. Coverage of a 100 m horizontal listening line extending from 0 to 100 m at a height of 0 m was analyzed. Source locations every 5 m in the rectangular grid were analyzed to evaluate coverage on the listening line. The analysis region only extends half way across the listening line because the nearest and farthest points swap from this point on. Air absorption was neglected.

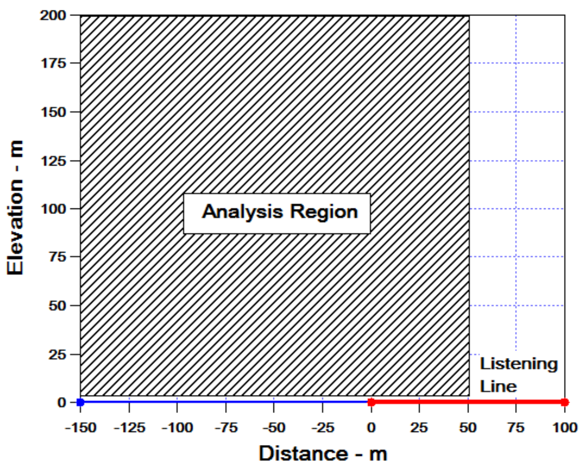

Fig. 7. Region covered to analyze possible source locations for CBT arrays. A listening line extends from 0 to 100 m horizontally at a height of 0 m (lower right). The 200 x 200 m analysis region extends horizontally from -150 to 50 m and vertically from 5 to 200 m. Data was analyzed at every point in the cross-hatched area with steps of 5 m.

4.1. SPL Error

To get an idea of which source locations in the rectangular grid are acceptable, the SPL error along the listening line was evaluated for each possible source location in the analysis region.

The SPL error is the difference between the maximum and minimum SPL values in dB along the listening line. For a CBT array source, the minimum SPL values always occurs at the ends of the covered region, while the SPL at intermediate points is always higher. Fig. 6 shows an example listening line SPL error of about 2.3 dB, i.e. the error is zero at the ends of the listening line and has a maximum deviation of +2.3 dB at an intermediate point.

Figure 8 shows the maximum SPL error in dB plotted in the analysis region of Fig. 7 in grey scale every 5 m. Errors over 8 dB were judged unacceptable and were not plotted. Constant SPL error contours for the data in Fig. 8 are plotted in Fig. 9. The plots show that the error rises dramatically for sources located near the listening line and decreases significantly for source locations far from the listening line, i.e. at the left and top of the analysis region.

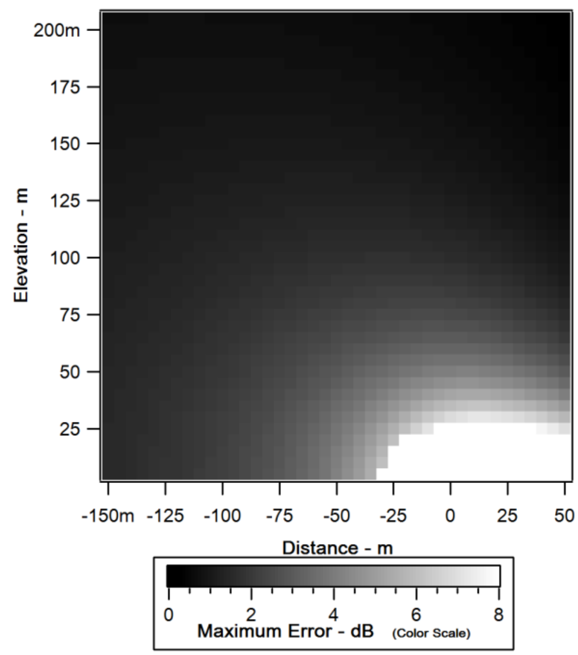

Fig. 8. Display of SPL error on rectangular grid for the analysis region displayed in Fig. 7. The data is displayed every 5 m in a grey color scale which covers the error range of 0 to 8 dB with black lowest and white highest.

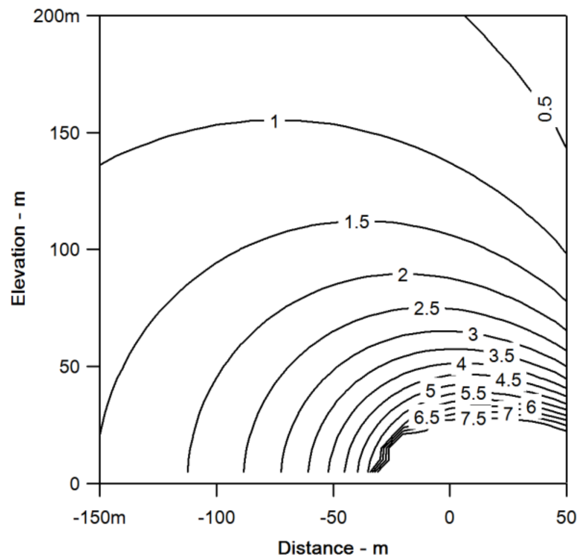

Fig. 9. SPL equal-error contour plots in dB for the data shown in Fig. 8. Note that the SPL error is very high for source locations close to the listening line (bottom right of the graph) and lowest for points farthest away (left and top of the graph).

In general, the error analysis shows that that source locations very close to the listening line exhibit high SPL errors and that locations far away exhibit low error. Selection of source locations is a balance between getting the source as close to the listening line as possible while simultaneously having an acceptable value of SPL error.

4.2. Near-Far Distance Ratio

The near-far distance ratio is the ratio between the distances from the source to the farthest point on the listening line divided by the distance to the nearest point. This ratio is significant because ratios beyond 4 to 1 cannot be easily corrected with the off-axis behavior of a practical CBT array. Simulations show that the off-axis behavior of a practical CBT array is not maintained consistently beyond the 12 dB down point from on axis.

Figure 10 shows the near-far distance ratios plotted in the analysis region shown in Fig. 7 in grey scale at every 5 m. Ratios between 1 and 5 are plotted. Correspondingly, Fig. 11 shows the constant-distance ratio contour plots for the grey scale data of Fig. 10. The plots show that the distance ratio increases quickly as the start of the listening line is approached.

Similar to the SPL error graphs of Fig. 8 and 9, the distance ratios between nearest and farthest points is highest for points close to the listening line and lowest for points far away.

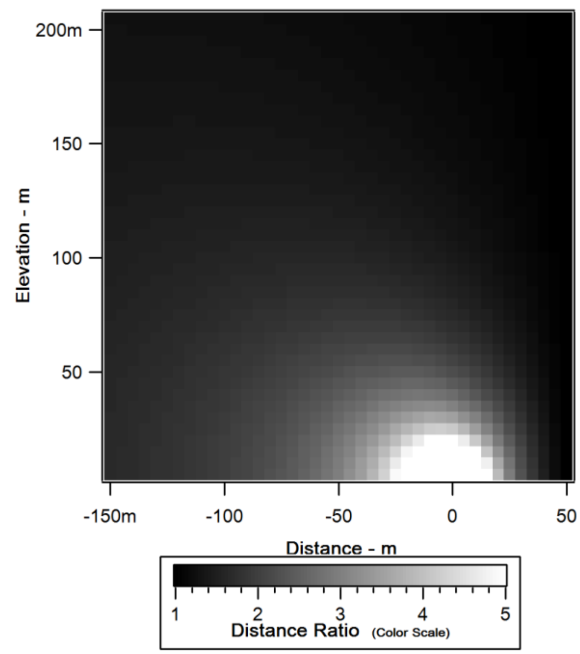

Fig. 10. Display of distance ratios on rectangular grid for the region displayed in Fig. 7. The data is displayed every 5 m in a grey color scale which covers the distance ratios of 1 to 5 with black lowest and white highest.

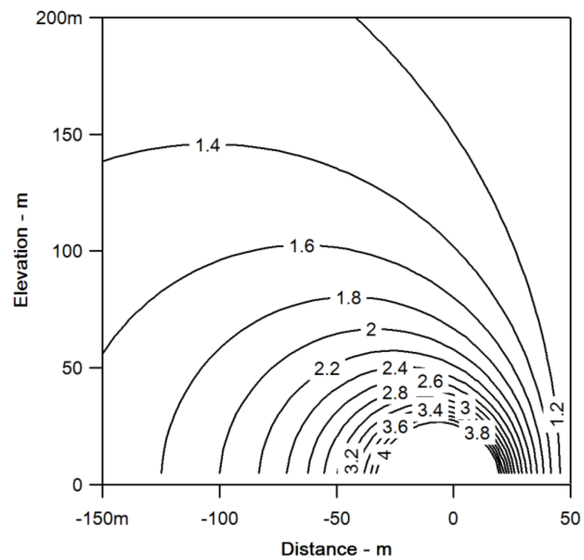

Fig. 11. Distance ratio contour plots for the grey scale data shown in Fig. 10.

4.3. Required Beamwidth

In order to cover the listening line accurately, the CBT source array height and arc angle must be chosen. The arc angle directly affects the beamwidth of the array's radiation.

The beamwidth of a CBT array is very significant because it is tied closely to the angle of the circular arc. The CBT array beamwidth is approximately equal to 75% of its arc angle. For an arc angle limit of 180°, this means that the beamwidth can be no higher than 135° (= 0.75 x 180°). This places a clear limit on source locations that require a beamwidth greater than 135° to properly cover the listening region.

Fig. 12 displays a grey scale plot of the required beamwidth at 5 m increments for the analysis region shown in Fig. 7. Fig. 13 shows the corresponding beamwidth contours for the grey scale data shown in Fig. 12.

These beamwidth plots show that as the array position is located over the start of the listening line (at 0 m), the required array vertical beamwidth becomes very large and in exceeds 120°! Correspondingly, for locations down in the lower left of the region where the array is quite low and its coverage essentially grazes the listening line, the required beamwidth values are very narrow in the range of 5° to 20°.

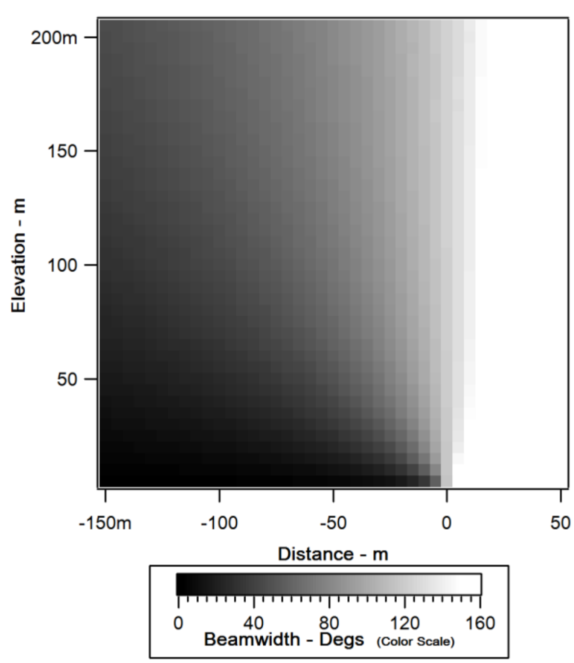

Fig. 12. Display of required beamwidth on rectangular grid for the region displayed in Fig. 7. The data is displayed in a grey color scale which covers the required beamwidth range of 0 to 160 degs with black lowest and white highest. The data is displayed every 5 m in a grey scale which covers beamwidths in the range of 0 (black) to 160° (white).

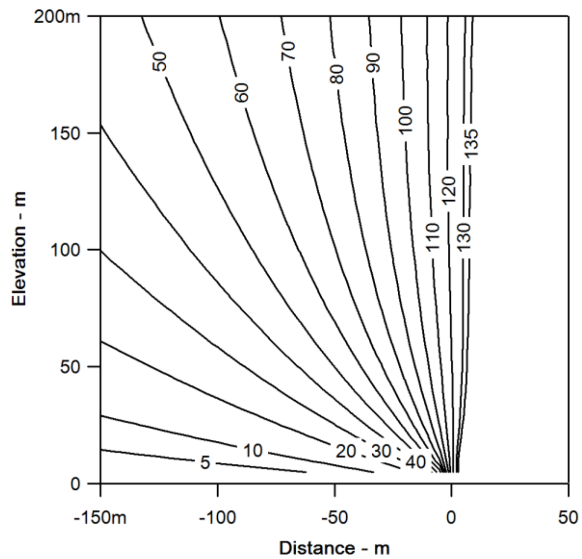

Fig. 13. Required beamwidth contour plots for the grey scale data shown in Fig. 12.

5. ACCEPTABLE ARRAY LOCATIONS

Considering the results of all three analyses methods that evaluate acceptable CBT array locations outlined in Section 4, i.e. SPL error, near-far distance ratio, and required beamwidth, the following graph illustrates all the acceptable array locations.

The following criteria were set for each of the analysis methods for acceptable array parameters:

1. SPL Error: $\leq$ 4 dB. Assumes an SPL error of about ±2 dB from front to back.
2. Near-Far Distance Ratio: $\leq$ 3.2. Assumes that the CBT lobe is accurate down to about 10 dB on the side of the coverage lobe.
3. Beamwidth: $10° \geq Angle \leq 120°$. This is a reasonable beamwidth range which results in a a minimum CBT arc angle of about 13° and a maximum of 160°.

With these criteria in mind, the grey region in Fig. 14 roughly illustrates all the acceptable array-mounting locations for the analysis region shown in Fig. 7.

Note that the boundary of the acceptable region is primarily set by only the SPL error and the beamwidth criteria. The acceptable near-far distance ratio criteria is outside the limits of the SPL error criteria.

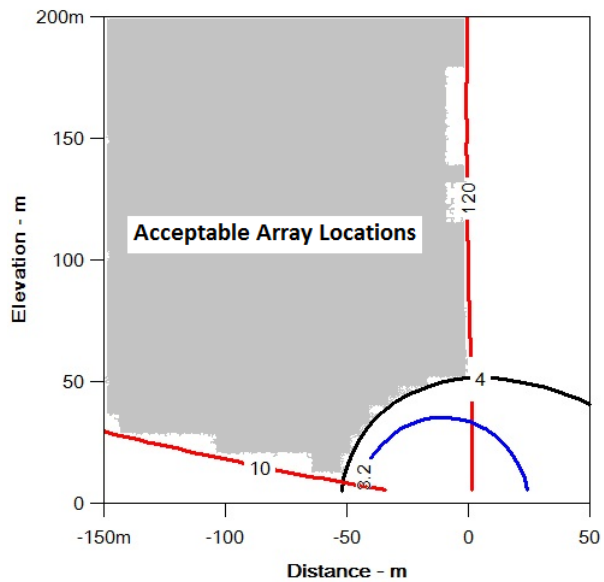

Fig. 14. Acceptable array locations (grey region) for the analysis region displayed in Fig. 7 for the chosen SPL error (black contour line), near-far distance ratio (blue contour line), and beamwidth criteria (red contour line). Note that valid region boundary is primarily set by the SPL error (black line) and the beamwidth criteria (red line).

6. DESIGN PROCEDURE

A typical design starts with several physical parameters specified including the length of the coverage region, the desired array mounting height and setback distance, and the desired lower beamwidth-control operating frequency limit.

The design is completed by calculating: the required array beamwidth and circular arc angle, the physical length (height) of the array, and the array downward tilt angle.

The following procedure outlines the steps required in a CBT array design for a specific coverage situation.

Desired Input Parameters:

1. Length of region to be covered (Coverage or listening line length),
2. Source mounting height,
3. Source mounting setback, and
4. Lower operating frequency limit.

Calculate:

1. Tilt angle to end of listening line,
2. Tilt angle to start of listening line,
3. Coverage angle for listening line,
4. Ratio of distances to start and end of listening line,
5. Array beamwidth (-6 dB),
6. Array circular arc angle, and
7. Array height.

6.1. Calculate End Tilt Angle

As stated before, the loudspeaker array is aimed at the farthest point or end of the listening line.

The downward tilt angle to the end of the listening line is given by:

$$\varphi_{EndTiltAngle} = -\tan^{-1}\left(\frac{D_{Height}}{D_{Offset} + D_{ListeningLine}}\right)$$

Where

$\varphi_{EndTiltAngle}$
$=$ Source tilt angle to end of listening line,
D_{Height} = Source height,
D_{Offset} = Source horizontal offset to start of listening line, and
$D_{ListeningLine}$ = Listening line length.

6.2. Calculate Start Tilt Angle

The downward tilt angle to the start of the listening line is given by:

$$\varphi_{StartTiltAngle} = -\tan^{-1}\left(\frac{D_{Height}}{D_{Offset}}\right)$$

Where

$\varphi_{StartTiltAngle}$ = Tilt angle to start of listening line,
D_{Height} = Source height, and
D_{Offset} = Source horizontal offset to start of listening line

6.3. Calculate Coverage Angle

The listening line coverage angle is the total angle as seen by the source that brackets the complete listening line. The coverage angle for the listening line is required to calculate the beamwidth of the array that correctly covers the listening line. It is simply just the absolute difference between the starting and ending tilt angles:

$$\emptyset_{CoverageAngle} = Abs[\emptyset_{StartTiltAngle} - \emptyset_{EndTiltAngle}]$$

6.4. Calculate Distance Ratio

The distance ratio is the ratio of the distances from the source to the farthest point of the coverage region to the nearest point of the coverage region. This ratio is required to calculate the required CBT array beamwidth to properly cover the desired region.

$$Ratio = \frac{\text{Distance to Farthest Point}}{\text{Distance to Nearest Point}} = \frac{D_{Far}}{D_{Near}}$$

Where

$$D_{Far} = \sqrt{{D_{Height}}^2 + (D_{Offset} + D_{ListeningLine})^2}$$

$$D_{Near} = \sqrt{{D_{Height}}^2 + {D_{Offset}}^2}$$

Where

D_{Height} = Source height,
D_{Offset} = Source horizontal offset to start of listening region, and
$D_{ListeningLine}$ = Listening line length.

6.5. Calculate Source Beamwidth

The CBT array beamwidth (-6 dB) must be chosen so that the SPL level at the start of the listening line is equal to the level at the end of the listening line. Because the array is aimed at the end of the listening line, the front of the line is illuminated by an off-axis point on the directional lobe of the array. The array's beamwidth must then be chosen so that this point on the coverage lobe has the correct level at the start of the listening line.

The directional lobe of a CBT array is fully defined by a Legendre function (Fig. 2 and also [7]). In order to calculate beamwidth, a value of the inverse Legendre function is required which can only be generated numerically by referring to a graph..

The following method requires the coverage angle of the array (Section 6.3), the distance ratio (Section 6.4), along with a graphical lookup of a value from the inverse Legendre function graph to determine the require beamwidth.

First, given the distance ratio (Section 6.4) look up a value for the "Normalized Half Angle" ($R_{NormHalfAngle}$) on the following graph:

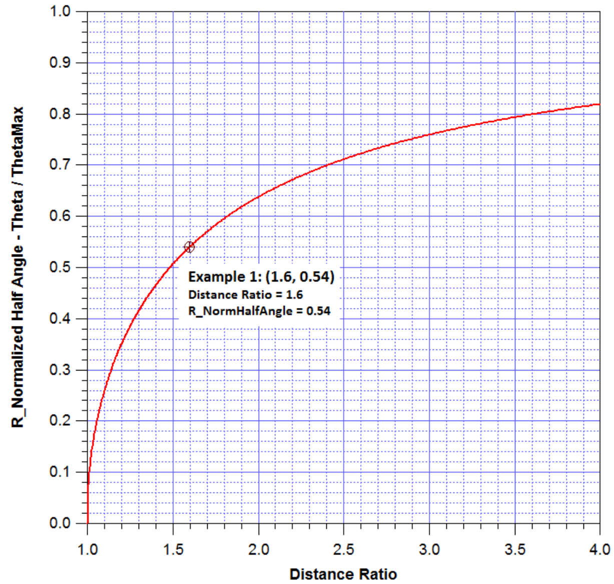

Fig. 15. Graph of inverse Legendre function to yield values for calculation of array beamwidth. A point on the graph (1.6, 0.54) is plotted for example #1 in Section 7.1..

The equation for beamwidth is then:

$$\emptyset_{BW} = \frac{1.276}{R_{NormHalfAngle}} \emptyset_{CoverageAngle}$$

The numerical value 1.276 is given without proof.

6.6. Calculate CBT Array Arc Angle

The beamwidth angle of a CBT array is approximately 75% of the array's arc angle. Solving for the arc angle yields:

$$\text{ArcAngle} = \frac{\emptyset_{BW}}{0.75}$$

6.7. Calculate CBT Array Height

The height of a CBT array is related to the array's beamwidth and lower beamwidth control frequency [7, Eq. 3]:

$$\text{ArrayHeight} = \frac{K}{\theta_{BW} f_L} = \frac{3.05\text{x}10^4}{\theta_{BW} f_L}$$

Where

$\emptyset_{BW}$ = Array beamwidth in Degs

f_L = Array lower beamwidth control frequency

$K = A$ constant=3.05x10^4 (meters-degs-Hz units)

7. EXAMPLE DESIGNS

This section presents two example CBT array designs:

1. **A Wide-Angle Near-Throw Design:** A broad-band wide-angle design covering a relatively short straight coverage line from a point directly above the start of the coverage line, and

2. **A Narrow-Angle Far-Throw Design:** A voice-only far-throw narrow-angle design, which covers a long low-ceiling space from a point, offset behind the start of the coverage line region.

7.1. Wide-Angle Near-Throw Design

This design example models a relatively short coverage region line of 25 m with a specified array location 20 m directly over the start of the coverage area. Wide-band coverage control down to 200 Hz is required for music and speech.

7.1.1. Desired Coverage Parameters:

1)	Coverage Line Length	= 25 m,
2)	Source Setback	= 0 m,
3)	Source Height	= 20 m, and
4)	Lower Operating Limit	= 100 Hz.

7.1.2. Calculate End Tilt Angle

The tilt angle to the start of the listening line is given by:

$$\emptyset_{TiltAngle} = -tan^{-1}\left(\frac{20}{25}\right) \cong -38.7°$$

7.1.3. Calculate Start Tilt Angle

The tilt angle to the start of the listening line is given by:

$$\varphi_{StartTiltAngle} = -\tan^{-1}\left(\frac{20}{0}\right) = -90°$$

7.1.4. Calculate Coverage Angle

The coverage angle is given by:

$$\emptyset_{CoverageAngle} = Abs[90° - 38.7°] = 51.3°$$

7.1.5. Calculate Distance Ratio:

The coverage region distance ratio is calculated by the dividing the distance from the source to the farthest coverage region (end) point by the distance from the source to the nearest coverage region (start) point:

$$D_{Far} = \sqrt{20^2 + 25^2 =} \; 32.02 \text{ m}$$

$$D_{Near} = 20 \text{ m}$$

$$\text{Ratio} = \frac{D_{Far}}{D_{Near}} = \frac{32.02}{20} = 1.60$$

7.1.6. Calculation of Source Beamwidth:

Knowing the array coverage angle and distance ratio, the beamwidth can be calculated using the graph of Fig. 15 and the equation in Section 6.5. The distance ratio value of 1.6 is used to look up a value of 0.54 for $R_{NormHalfAngle}$ in the graph of Fig. 15.

The source beamwidth is then calculated using the formula in Section 6.5.

$$\emptyset_{BW} = \frac{1.276}{R_{NormHalfAngle}}\emptyset_{CoverageAngle} \cong \frac{1.276}{0.54}51.3° \cong 121°$$

7.1.7. Calculation of CBT Array Arc Angle:

$$\text{ArcAngle} = \frac{\emptyset_{BW}}{0.75} \cong \frac{121.1}{0.75} \cong 161.4 \text{ Degs}$$

7.1.8. Calculation of Array Height:

$$\text{ArrayHeight} = \frac{K}{\theta f_l} = \frac{3.05\text{x}10^4}{\theta f_l} \cong \frac{3.05\text{x}10^4}{121.1(100)} \cong 2.5 \text{ m (100 in)}$$

7.1.9. Simulated Array

A CBT array was designed to generate the following frequency response graphs. The array is 2.5 m tall, has an arc angle of 160°, is composed of 115 point sources spaced at about 32 mm (1.25") center to center, and uses continuous Legendre shading truncated at 12 dB down.

7.1.10. Coverage Pictorial:

Fig. 16 shows a pictorial of this coverage situation.

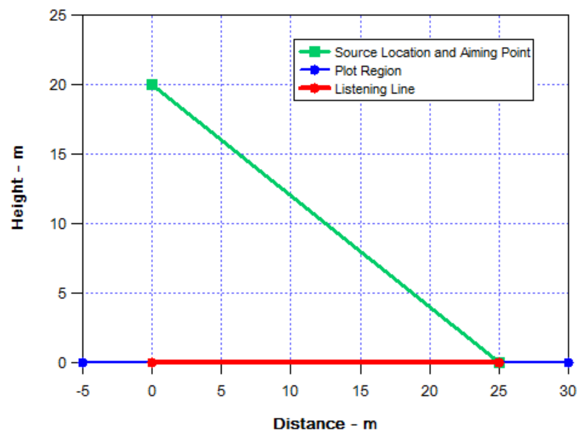

Fig. 16. Coverage line and source location for the wide-angle near-throw example. The coverage line is 25 m long and is covered by CBT source array that is located 20 m above the start of the coverage line. The array is aimed down at an angle of about 39° aimed directly at the rear of the coverage line.

7.1.11. Raw Un-Equalized Frequency Response vs. Distance

Fig. 17 shows the simulated un-equalized frequency responses of this CBT array at six locations along the coverage line in increments of 5 m. The inherent 3 dB/octave power rolloff of the array's coverage is clearly indicated above 100 Hz ([7] where the power rolloff is called "On-axis loss").

The power rolloff of the curves commences about 100 Hz approximately meeting the desired lower operating limit. Also note the chaotic response above 10 kHz due to the finite 1.25" spacing of the point sources.

Note that the each of the response curves are very similar to each other and are essentially at the same level.

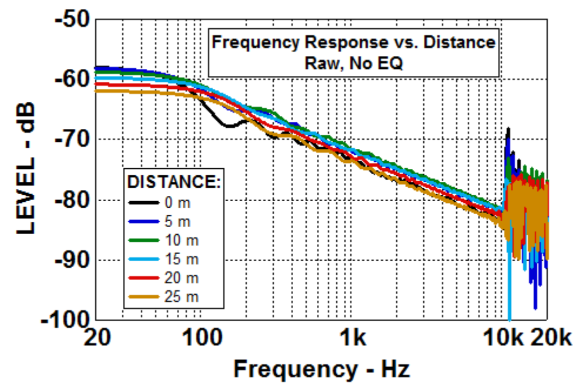

Fig. 17. Raw un-equalized frequency responses at six equally-spaced locations along the listening line for the wide-angle near-throw CBT example design. Note the 3 dB per octave or 10 dB per decade falling power response (above 100 Hz) that is a characteristic of all circularly curved sources. This response rolloff must be equalized (see equalized curves in the next figure).

7.1.12. Equalized Frequency Response vs. Distance

Fig. 18 shows the simulated equalized frequency response of this CBT array at six locations along the coverage line in increments of 5 m.

The response was equalized flat at 10 m (green curve) from the start of the 25 m listening line. This midpoint position is the approximate location where the maximum SPL error is exhibited. From 400 Hz to 8 kHz the curves fit a very-tight envelope of about 2.5 dB.

Note the extreme uniformity of the curves.

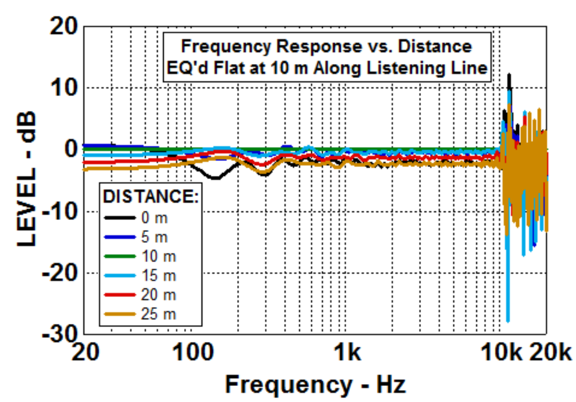

Fig. 18. Equalized frequency responses at six equally-spaced locations along the listening line for the wide-angle near-throw CBT example design. The responses are equalized at a point 10 m or 40% from the start of the 25 m listening line. The end point response curves (0 m black and 25 m orange) illustrate that the frequency responses are very similar and equal in level (by design!).

7.2. Narrow-Angle Far-Throw Design

This design example models a long coverage region line of 100 m with a specified array location 10 m high and setback 15 m before the start of the coverage area. Limited coverage control down to 400 Hz is adequate for speech reproduction.

7.2.1. Desired Coverage Parameters:

1) Coverage Line Length = 100 m,
2) Source Setback = -40 m,
3) Source Height = 15 m, and
4) Lower Operating Limit = 400 Hz.

7.2.2. Calculate End Tilt Angle:

The source downward tilt angle to the end of the listening line is given by:

$$\emptyset_{TiltAngle} = -tan^{-1}\left(\frac{15}{40+100}\right) \cong -6.1 \text{ Degs}$$

7.2.3. Calculate Start Tilt Angle

The tilt angle to the start of the listening line is given by:

$$\varphi_{StartTiltAngle} = -\tan^{-1}\left(\frac{15}{40}\right) = -20.56°$$

7.2.4. Calculate Coverage Angle

The coverage angle is given by:

$$\emptyset_{CoverageAngle} = Abs[20.56° - 6.1°] = 14.46°$$

7.2.5. Calculate Distance Ratio:

The coverage region distance ratio is calculated as follows:

$$D_{Far} = \sqrt{15^2 + 140^2} \cong 140.8 \text{ m}$$

$$D_{Near} = \sqrt{15^2 + 40^2} \cong 42.7 \text{ m}$$

$$\text{DistanceRatio} = \frac{D_{Far}}{D_{Near}} = \frac{140.8}{42.7} \cong 3.3$$

7.2.6. Calculation of Source Beamwidth:

Knowing the array coverage angle and distance ratio, the beamwidth can be calculated using the graph of Fig. 15 and the equation in Section 6.5. The distance ratio value of 3.3 is used to look up a value of 0.781 for RNormHalfAngle in the graph of Fig. 15.

The source beamwidth is then

$$\emptyset_{BW} = \frac{1.276}{R_{NormHalfAngle}} \emptyset_{CoverageAngle} \cong \frac{1.276}{0.781} 14.46° \cong 23.6°$$

7.2.7. Calculation of CBT Array Arc Angle:

$$\text{ArcAngle} = \frac{\emptyset_{BW}}{0.75} \cong \frac{23.6°}{0.75} \cong 31.5 \text{ Degs}$$

7.2.8. Calculation of Array Height:

$$\text{ArrayHeight} = \frac{K}{\theta f_l} = \frac{3.05\text{x}10^4}{\theta f_l} = \frac{3.05\text{x}10^4}{23.6(400)} \cong 3.23 \text{ m (127.2 in)}$$

7.2.9. Simulated Array

A CBT array was designed to generate the following frequency response graphs. The array is 3.23 m tall, has an arc angle of about 31.5°, is composed of 60 point sources spaced at about 56 mm (2.2") center to center, and uses continuous Legendre shading truncated at 12 dB down.

7.2.10. Coverage Pictorial:

Fig. 19 shows a pictorial of this coverage situation.

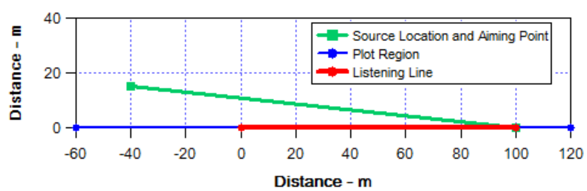

Fig. 19. Coverage line and source location for the narrow-angle far-throw example. The coverage line is 100 m long and is covered by CBT source array that is located only 15 m above and offset 40 m behind the start of the coverage line. The array is aimed down at an angle of about 6° aimed directly at the rear of the coverage line.

7.2.11. Raw Un-Equalized Frequency Response vs. Distance

Fig. 20 shows the simulated un-equalized frequency responses of this CBT array at six locations along the coverage line equally spaced every 20 m. The inherent 3 dB/octave power rolloff of the array's coverage is clearly indicated above 200 to 400 Hz.

Note the chaotic response above 11 kHz due to the finite 2.2" spacing of the point sources.

Note that the each of the response curves between 600 Hz and 10 kHz are very similar to each other but separated somewhat in level.

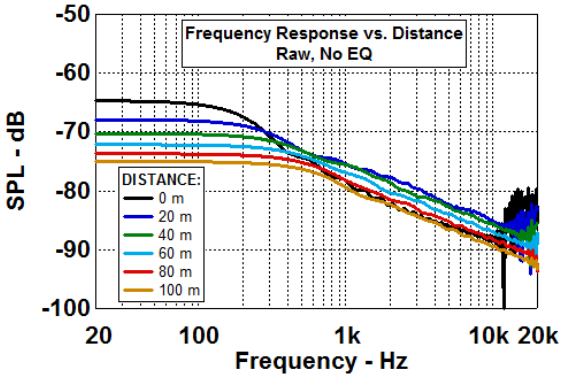

Fig. 20. Raw un-equalized frequency responses at six equally-spaced locations along the listening line for the wide-angle near-throw CBT example design. Note the 3 dB per octave or 10 dB per decade falling power response (above 300 Hz) that is a characteristic of all circularly curved sources. This response rolloff must be equalized (see equalized curves in the next figure).

7.2.12. Equalized Frequency Response vs. Distance

Fig. 21 (next page) shows the simulated equalized frequency response of this CBT array at six locations along the coverage line in increments of 20 m.

The response was equalized flat at 40 m (green curve) from the start of the 100 m listening line. This midpoint position is the approximate location where the maximum SPL error is exhibited. From 400 Hz to 10 kHz the curves fit a fairly-tight envelope of about 4 dB.

Note the extreme uniformity of the curves.

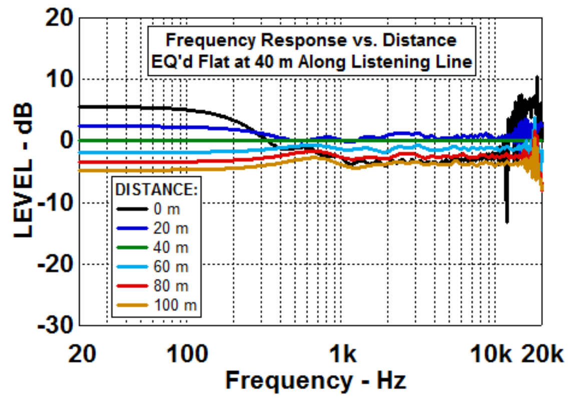

Fig. 21. Equalized frequency responses at six equally-spaced locations along the listening line for the wide-angle near-throw CBT example design. The responses are equalized at a point 40 m or 40% from the start of the listening line.

7.2.13. Equalized and Normalized Frequency Response vs. Distance

Fig. 22 shows the simulated equalized and 1 kHz normalized frequency responses of the far-throw array at six locations along the coverage line in increments of 20 m. Between 500 Hz and 10 kHz the response curves are extremely uniform and fit a very-close envelope of about 1.5 dB!

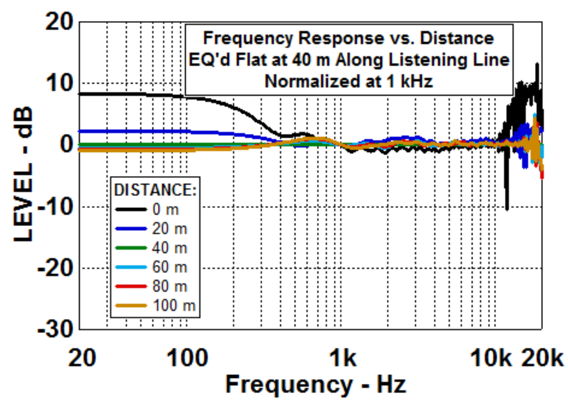

Fig. 22. Equalized and normalized frequency responses at six equally-spaced locations along the listening line for the narrow-angle far-throw CBT example design. The responses are equalized at a point 40 m or 40% from the start of the 100 m listening line and are normalized at 1 kHz.

7.2.14. Equalized Frequency Response vs. Distance to Illustrate SPL Error

Fig. 23 shows the simulated equalized frequency responses of the far-throw array at three locations along the coverage line including the end points at 0 and 100 m, and an intermediate point at 40 m where the response is equalized flat. These locations were selected to emphasize the maximum SPL error of the frequency response curves.

This graph illustrates the highest and lowest frequency responses for the design and the maximum SPL error deviation. The end-point response curves clearly illustrate that the frequency responses are very similar and equal in level (by design!).

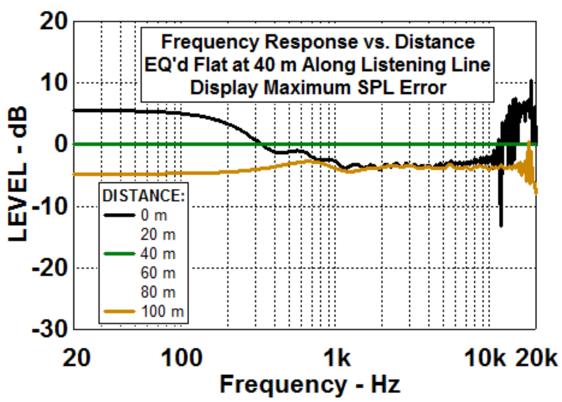

Fig. 23. Equalized frequency responses at three locations along the listening line for the wide-angle near-throw CBT example design. The responses are equalized at a point 40 m or 40% from the start of the 100 m listening line. These locations were selected to emphasize the maximum SPL error of the frequency response curves. The points include the start and end of the listening line and the point at which the curves are equalized flat. The end-point response curves illustrate that the frequency responses are very similar and equal in level (by design!). The chosen midpoint location is the approximate location where the maximum SPL error of about 4 dB is exhibited.

8. CONCLUSIONS

This paper presented guidelines for the design of free-standing CBT line arrays for sound reinforcement. Due to the extremely uniform coverage of CBT line arrays with frequency and distance, and their inherent constant-directivity characteristics, the design guidelines are much simplified compared to traditional loudspeaker arrays.

The arrays can easily be designed on paper without needing the complicated and expensive aiming and acoustic modeling software often required to optimize the coverage in a particular sound reinforcement situation.

The paper's guidelines depend heavily on the extreme uniformity of a CBT line array's vertical coverage with frequency. The CBT's vertical coverage beam is so uniform with frequency that points even 10 to 12 dB down on the sides of the beam are usable for even coverage.

The design method aims the array at the rear of the coverage region and then sets the beamwidth of the array to provide the same SPL level and frequency response coverage at the front of the region. Intermediate points from front to rear are then covered by the inherent well-behaved off-axis rolloff of the CBT array's main beam.

9. REFERENCES

[1] AFMG - Ahnert Feistel Media Group (http://www.afmg.eu/index.php/products.html). Also see Renkus-Heinz, Inc (http://www.renkus-heinz.com/ease/index.html).

[2] Odeon A/S (http://www.odeon.dk/).

[3] Bose Professional Systems Division (http://pro.bose.com/ProController?url=/pro/products/modeler66/modeler66.jsp).

[4] JBL Professional (http://www.jblpro.com/VTCalculator).

[5] A. Thompson, "Improved Methods for Controlling Touring Loudspeaker Arrays," 127th Convention of the Audio Engineering Society, Convention paper 7828 (Oct. 2009).

[6] M. Terrell and M Sandler, "Optimizing the Controls of Homogeneous Loudspeaker Arrays," 129th Convention of the Audio Engineering Society, Convention paper 8159 (Nov. 2010).

[7] D. B. Keele, Jr., "The Application of Broadband Constant Beamwidth Transducer (CBT) Theory to Loudspeaker Arrays," 109th Convention of the Audio Engineering Society, Convention paper 5216 (Sept. 2000).

[8] D. B. Keele, Jr., "Implementation of Straight-Line and Flat-Panel Constant Beamwidth Transducer (CBT) Loudspeaker Arrays Using Signal Delays," 113th Convention of the Audio Engineering Society, Convention paper 5653 (Oct. 2002).

[9] D. B. Keele, Jr., "The Full-Sphere Sound Field of Constant Beamwidth Transducer (CBT)

Loudspeaker Line Arrays," J. Aud. Eng. Soc., vol. 51, no. 7/8., pp. 611-624 (July/August 2003).

[10] D. B. Keele, Jr., "Practical Implementation of Constant Beamwidth Transducer (CBT) Loudspeaker Circular-Arc Line Arrays," presented at the 115th Convention of the Audio Engineering Society, New York, Convention paper 5863 (Oct. 2003).

[11] D. B. Keele, Jr. and D. J. Button, "Ground-Plane Constant Beamwidth Transducer (CBT) Loudspeaker Circular-Arc Line Arrays." presented at the 119th Convention of the Audio Engineering Society, Convention paper 6594 (Oct. 2005).

[12] D. B. Keele, Jr. "A Performance Ranking of Seven Different Types of Loudspeaker Line Arrays.' presented at the 129th Convention of the Audio Engineering Society, Convention paper 8155 (Nov. 2010).

[13] Xulie Feng, Yong Shen, D. B. Keele Jr., and Jie Xia, " Directivity-Customizable Loudspeaker Arrays Using Constant-Beamwidth Transducer (CBT) Overlapped Shading," presented at the 139th Convention of the Audio Engineering Society, New York, (Oct.-Nov. 2015).

[14] D. B. Keele, Jr., "Time/Phase Behavior of Constant Beamwidth Transducer (CBT) Circular-Arc Loudspeaker Line Arrays," presented at the 139th Convention of the Audio Engineering Society, New York, (Oct.-Nov. 2015).

[15] D. B. Keele, Jr., "Implementation of Segmented Circular-Arc Constant Beamwidth Transducer (CBT) Loudspeaker Arrays," presented at the 139th Convention of the Audio Engineering Society, New York, (Oct.-Nov. 2015).

[16] P. H. Rogers, and A. L. Van Buren, "New Approach to a Constant Beamwidth Transducer," J. Acous. Soc. Am., vol. 64, no. 1, pp. 38-43 (1978 July).

[17] "Huygens–Fresnel Principle," From Wikipedia, the free encyclopedia: (http://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle).